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AN ASYMPTOTICAL METHOD FOR DETERMINING THE COEFFICIENT OF HYDRAULIC RESISTANCE IN GAS-LIFT PROCESS BY THE LINES METHOD*

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Abstract. In the paper the identification problem is considered to determine the coefficient of hydraulic resistance in gas-lift process by the lines method. There exists a small parameter in the right side of the partial differential equation of hyperbolic type which is the inverse of the well depth *l*. Using the lines method the system of partial differential equations of hyperbolic type is reduced to the system of linear ordinary differential equations with respect to the volume of gas, gas liquid mixture (GLM) and their pressure. An asymptotical solution of the given equation is obtained and this solution is calculated at the point 2*l* (debit). On a concrete example is shown that the value of the coefficient of hydraulic resistance obtained by the asymptotical method differs from the real value of the coefficient of hydraulic resistance to the order 10^{-3}.

Keywords: Identification problem, lines method, the coefficient of hydraulic resistance, gas liquid mixture.

AMS Subject Classification: 49J15, 49J35.

1. Introduction.

The system of partial differential equations of hyperbolic type describing the motions of the gas and gas liquid mixture in the tubes $z=\frac{x}{|2L|}=\varepsilon x$ is denoted, when a small parameter ε is considered as the inverse of the well ($\varepsilon=\frac{1}{|2L|}$) depth l ($\varepsilon=\frac{1}{|2L|}$) [1, 7,13,15,21]:

$$\begin{cases} \dfrac{\partial P}{\partial t}=-\dfrac{c^2}{F}\dfrac{\partial Q}{\partial z}\varepsilon, \\ \dfrac{\partial Q}{\partial t}=-F\dfrac{\partial P}{\partial z}\varepsilon-2aQ, \end{cases} \tag{1}$$

where $P=P(x,t)-$ is pressure, $Q(x,t)$- is volume of gas and GLM, correspondingly, c- is the speed of sound in the gas and GLM, correspondingly; $2a=\frac{g}{\omega_c}+\frac{\lambda_c\omega_c}{2D}$; g, λ_c- acceleration of gravity and hydraulic resistance in the

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gas and GLM, correspondingly; ω_c - averaged over the cross section of the velocity of the gas and mixture in the annular zone and lift, internal diameters of lift and annular space, which are constant over the axes, F - cross -sectional area of tubing constant on axes. If we denote $l = \frac{1}{2n}$ we obtain from (1) approximately linear model [8, 20] applying the lines method for n=2

$$\begin{cases} \dot{P}_1 = -\frac{c_1^2 \varepsilon}{F_1 l} Q_1 + \frac{c_1^2 \varepsilon}{F_1 l} Q_0, \\ \dot{Q}_1 = -\frac{F_1 \varepsilon}{l} P_1 + \frac{F_1 \varepsilon}{l} P_0 - 2a_1 Q_1, \\ \dot{P}_2 = -\frac{c_2^2 \varepsilon}{F_2 l} Q_2 + \frac{c_2^2 \varepsilon}{F_2 l} Q_1 + \frac{c_2^2 \varepsilon}{F_2 l} Q_{pl}, \\ \dot{Q}_2 = -\frac{F_2 \varepsilon}{l} P_2 + \frac{F_2 \varepsilon}{l} P_1 - 2a_2 Q_2 + \frac{F_2 \varepsilon}{l} P_{pl}, \end{cases} \tag{2}$$

where Q_{pl}, P_{pl} are volume flow and pressure of the reservoir at the bottom of the well.

After some transformations the system (2) is reduced to [11, 19]

$$\dot{x} = (A_0(\lambda_c) + A_1 \varepsilon)x + B\varepsilon u + V\varepsilon, \tag{3}$$

with initial condition

$$x_0 = \left[P_1^0, Q_1^0, P_2^0, Q_2^0\right]',$$

where x_0 is the initial position of the well and

$$A_1 = \begin{bmatrix} 0 & -\frac{c_1^2}{F_1 l} & 0 & 0 \\ -\frac{F_1}{l} & 0 & 0 & 0 \\ 0 & \frac{c_2^2}{F_2 l} & 0 & -\frac{c_2^2}{F_2 l} \\ \frac{F_2}{l} & 0 & -\frac{F_2}{l} & 0 \end{bmatrix}, \quad A_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -2a_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2a_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & \frac{c_1^2}{F_1 l} \\ \frac{F_1}{l} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$V = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{c_2^2}{F_2 l} \\ \frac{F_2}{l} & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{pl} \\ Q_{pl} \end{bmatrix}, \quad u = \begin{bmatrix} P_0 \\ Q_0 \end{bmatrix}, \quad x = \left[P_1, Q_1, P_2, Q_2\right]'.$$

2. Problem statement. Let's have some statistics that at the given initial volumes of gas $\tilde{Q}_0^i$ the debit $\tilde{Q}_2^i$ is measured at the output, i.e. $\tilde{Q}_0^i$, and $\tilde{Q}_2^i$ are known ($i = \overline{1, N}$), where N is the number of tests.

It is required to find such values of the coefficient of hydraulic resistance λ_c [3, 4, 12, 18], which system (3) will describe the motion of GLM in the lift, closer to practice (adequate mathematical model). To solve this problem, the objective function based on the lowest square deviation from the real initial data $\tilde{Q}_2^i$ is constructed, i.e. it is required to minimize the functional

$$f(\lambda_c) = \sum_{i=1}^{N} \left[\tilde{Q}_2^i - Q_2^i\right]^2 . \tag{4}$$

The analogues problem one can consider for discrete Roesser equation [2].

Thus, the problem of determining the coefficient of hydraulic resistance λ_c can be reduced to finding the gradient of functional (4) [5,6, 9,10, 17].

The general solution of equation (3) can be represented as follows

$$\begin{aligned} x(t) &= e^{(A_0 + A_1\varepsilon)t} x_0 + \varepsilon \int_0^t e^{(A_0 + A_1\varepsilon)(t-\sigma)} Bud\sigma + \varepsilon \int_0^t e^{(A_0 + A_1\varepsilon)(t-\sigma)} Vd\sigma \approx \\ &\approx (E + A_0 t + \frac{t^2}{2} A_0^2) x_0 + \varepsilon((A_1 t + \frac{t^2}{2} A_0 A_1 + \frac{t^2}{2} A_1 A_0) x_0 + \\ &+ (Et + \frac{t^2}{2} A_0 + \frac{t^3}{6} A_0^2)(Bu + V)) = x_1 + \varepsilon x_2, \end{aligned} \tag{5}$$

where

$$x_1 = (E + A_0 t + \frac{t^2}{2} A_0^2) x_0,$$

$$x_2 = (A_1 t + + \frac{t^2}{2} A_0 A_1 + \frac{t^2}{2} A_1 A_0) x_0 + (Et + \frac{t^2}{2} A_0 + \frac{t^3}{6} A_0^2)(Bu + V).$$

After the following transformation

$$A_0 = A_2 + \lambda_c A_3,$$

where

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -2a_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{g}{\omega_c} \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dfrac{\omega_c}{2D} \end{bmatrix},$$

the solution (5) is represented as follows

$$x(t) = (E + (A_2 + \lambda_c A_3)t + \frac{t^2}{2}(A_2 + \lambda_c A_3)^2)x_0{}' + \varepsilon((A_1 t + \frac{t^2}{2}(A_2 + \lambda_c A_3)A_1 + \tag{6}$$
$$+\frac{t^2}{2}A_1(A_2 + \lambda_c A_3))x_0{}' + (Et + \frac{t^2}{2}(A_2 + \lambda_c A_3) + \frac{t^3}{6}(A_2 + \lambda_c A_3)^2)(Bu + V)).$$

From (6) Q_2 has the form

$$Q_2 = J \cdot x(t), \tag{7}$$

where $J = [0,\ 0,\ 0,\ 1]$, E- is the identity matrix of the corresponding dimension.

Then, substituting (6) and (7) in (4) we have

$$f(\lambda_c) = \sum_{i=1}^{N}\left(\tilde{Q}_2^i - Q_2^i\right)^2 = \sum_{i=1}^{N}\left(\tilde{Q}_2^i - Jx(T)\right)^2 = \sum_{i=1}^{N}\left(\tilde{Q}_2^i - Jx_1 - \varepsilon J x_2\right)^2. \tag{8}$$

$f(\lambda_c)$ from (8) takes the following form throwing the members in which the degree of ε is greater than 1

$$f(\lambda_c) = \sum_{i=1}^{N}\left(\tilde{Q}_2^i - Jx_1\right)^2 - 2\varepsilon\sum_{i=1}^{N}\left(\tilde{Q}_2^i - Jx_1\right)Jx_2 = f_1(\lambda_c) + \varepsilon f_2(\lambda_c) \tag{9}$$

with accuracy $O(\varepsilon^2)$, where

$$\begin{cases} f_1(\lambda_c) = \sum_{i=1}^{N}\left(\tilde{Q}_2^i - Jx_1\right)^2 = \\ = \sum_{i=1}^{N}\left[\tilde{Q}_2^i - J(E + (A_2 + \lambda_c A_3)T + \frac{T^2}{2}(A_2 + \lambda_c A_3)^2)x_0^i{}'\right]^2, \\ f_2(\lambda_c) = -2\sum_{i=1}^{N}\left(\tilde{Q}_2^i - Jx_1\right)Jx_2 = \\ = -2\sum_{i=1}^{N}\left(\tilde{Q}_2^i - J(E - (A_2 + \lambda_c A_3)T + \frac{T^2}{2}(A_2 + \lambda_c A_3)^2)x_0^i{}'\right)J \times \\ \times ((A_1 T + \frac{T^2}{2}(A_2 + \lambda_c A_3)A_1 + \frac{T^2}{2}A_1(A_2 + \lambda_c A_3))x_0^i{}' + (TE + \frac{T^2}{2}(A_2 + \lambda_c A_3) + \\ + \frac{T^3}{6}(A_2 + \lambda_c A_3)^2)(Bu + V)). \end{cases} \tag{10}$$

To solve the initial optimization problem we find the gradient of the functional $f(\lambda_c)$ as

$$\frac{\partial f}{\partial \lambda_c} = \left(\frac{\partial f_1}{\partial \lambda_c} + \frac{\partial f_2}{\partial \lambda_c}\varepsilon\right)\cdot \delta\lambda$$

and consider the equation

$$\frac{\partial f(\lambda_c)}{\partial\lambda_c}=0, \tag{11}$$

for satisfying the last equation we have the following relations

$$\frac{\partial f_1(\lambda_c)}{\partial\lambda_c}=0,\ \frac{\partial f_2(\lambda_c)}{\partial\lambda_c}=0. \tag{12}$$

The relation (9) allows one to solve the system of algebraic equations (12) analytically, if λ_c sought in the form [14, 16]

$$\lambda_c=\lambda_0+\varepsilon\lambda_1+...,$$

i.e. calculating the equation $\frac{\partial f_1(\lambda_c)}{\partial\lambda_c}=0$ we have

$$\frac{\partial f_1(\lambda_c)}{\partial\lambda_c}=-2\sum_{i=1}^{N}[\tilde{Q}_2^i-J(E+A_2T+\lambda_cA_3T+\frac{T^2}{2}A_2^2+\frac{T^2}{2}\lambda_cA_2A_3+\frac{T^2}{2}\lambda_cA_3A_2+ \tag{13}$$

$$+\frac{T^2}{2}\lambda_c^2A_3^2)x_0^{i\,\prime}]J(A_3T-T^2A_2A_3-T^2\lambda_cA_3^2)x_0^{i\,\prime}=0.$$

After some transformations in the equation (13) we obtain the following relation for λ_0

$$\lambda_0^3\sum_{i=1}^{N}J\frac{T^2}{2}A_3^2J'x_0^iT^2A_3^2x_0^{i\,\prime}+\lambda_0^2-\left(\sum_{i=1}^{N}J(A_3T+\frac{T^2}{2}A_2A_3+\frac{T^2}{2}A_3A_2)J'x_0^i\,T^2A_3^2x_0^{i\,\prime}-\right.$$

$$\left.-J\frac{T^2}{2}A_3^2J'x_0^i(A_3T-T^2A_2A_3)x_0^{i\,\prime}\right)+\lambda_0\left(\sum_{i=1}^{N}J(E+A_2T+\frac{T^2}{2}A_2^2)J'x_0^i\,T^2A_3^2x_0^{i\,\prime}-\right. \tag{14}$$

$$\left.-J(A_3T+\frac{T^2}{2}A_2A_3+\frac{T^2}{2}A_3A_2)J'x_0^i(A_3T-T^2A_2A_3)x_0^{i\,\prime}-\tilde{Q}_2^i\,JT^2A_3^2x_0^{i\,\prime}\right)+$$

$$+\left(\sum_{i=1}^{N}\tilde{Q}_2^iJ(A_3T-T^2A_2A_3)x_0^{i\,\prime}-J(E+A_2T+\frac{T^2}{2}A_2^2)J'x_0^i(A_3T-T^2A_2A_3)x_0^{i\,\prime}\right)=0.$$

Let's calculate $\frac{\partial f_2(\lambda_c)}{\partial\lambda_c}=0$

$$\frac{\partial f_2(\lambda_c)}{\partial\lambda_c}=-2\sum_{i=1}^{N}\left((JA_3T-JT^2(A_2+\lambda_cA_3)A_3)x_0^{i\,\prime}\right)J((A_1T+\frac{T^2}{2}(A_2+\lambda_cA_3)A_1+$$

$$+\frac{T^2}{2}A_1(A_2+\lambda_cA_3))x_0^{i\,\prime}+(TE+\frac{T^2}{2}(A_2+\lambda_cA_3)+\frac{T^3}{6}(A_2+\lambda_cA_3)^2)(Bu+V))-$$

$$-2\sum_{i=1}^{N}\left(\tilde{Q}_2^i-J(E-(A_2+\lambda_cA_3)T+\frac{T^2}{2}(A_2+\lambda_cA_3)^2)x_0^{i\,\prime}\right)J\times$$

$$\times((\frac{T^2}{2}A_3A_1+\frac{T^2}{2}A_1A_3)x_0^{i\,\prime}+(\frac{T^2}{2}A_3+\frac{T^3}{3}(A_2+\lambda_c A_3)A_3)(Bu+V)). \tag{15}$$

From (15) we may define λ_1 as follows

$$-\sum_{i=1}^{N}\left((JA_3T-\lambda_0 JT^2A_2A_3-\varepsilon\lambda_1 JT^2A_3^2)x_0^{i\,\prime}\right)J((A_1T+\frac{T^2}{2}A_2A_1+\lambda_0\frac{T^2}{2}A_3A_1+$$

$$+\varepsilon\lambda_1\frac{T^2}{2}A_3A_1+\frac{T^2}{2}A_1A_2+\lambda_0\frac{T^2}{2}A_1A_3+\varepsilon\lambda_1\frac{T^2}{2}A_3A_1)x_0^{i\,\prime}+(TE+\frac{T^2}{2}A_2+\lambda_0\frac{T^2}{2}A_3+$$

$$+\varepsilon\lambda_1\frac{T^2}{2}A_3+\frac{T^3}{6}A_2^2+\lambda_0\frac{T^3}{6}A_2A_3+\varepsilon\lambda_1\frac{T^3}{6}A_2A_3+\lambda_0\frac{T^3}{6}A_3A_2+\varepsilon\lambda_1\frac{T^3}{6}A_3A_2+$$

$$+\frac{T^3}{6}\lambda_0^2A_3^2+\frac{T^3}{3}\varepsilon\lambda_0\lambda_1A_3^2)(Bu+V))-$$

$$-\sum_{i=1}^{N}(\widetilde{Q}_2^i-J(E-A_2T-\lambda_0A_3T-\varepsilon\lambda_1A_3T+\frac{T^2}{2}\lambda_0A_2A_3+\varepsilon\lambda_1\frac{T^2}{2}A_2A_3+\lambda_0\frac{T^2}{2}A_3A_2+$$

$$+\varepsilon\lambda_1\frac{T^2}{2}A_3A_2+\lambda_0^2\frac{T^2}{2}A_3^2+\varepsilon\lambda_0\lambda_1T^2A_3^2+\varepsilon^2\lambda_1^2A_3^2)x_0^{i\,\prime})J((\frac{T^2}{2}A_3A_1+\frac{T^2}{2}A_1A_3)x_0^{i\,\prime}+$$

$$+(\frac{T^2}{2}A_3+\frac{T^3}{3}A_2A_3+\lambda_0\frac{T^3}{3}A_3^2+\varepsilon\lambda_1\frac{T^3}{3}A_3^2)(Bu+V))=0. \tag{16}$$

Thus, we formulate the following algorithm for finding λ_c.

Algorithm.

1. Entering the initial data and parameters from (3);
2. Entering the x_0, the statistical data $\widetilde{Q}_0^i$ and observations $\widetilde{Q}_2^i$ from the practice for the same well;
3. Finding $x(t)$ from (6);
4. Forming the functional $f(a_2(\lambda_c))$ from (9), i.e. finding $f_1(\lambda_c)$ and $f_2(\lambda_c)$ from (10)
5. Finding λ_0 and λ_1 from (14) , (16) to define the solution of the equation (11)
6. For sufficiently small number ε we check the condition $\left|\frac{\partial f(a_2(\lambda_c))}{\partial a_2(\lambda_c)}\right|<\varepsilon$: if it is satisfied, the calculations stop, otherwise go to step 2.

Now consider the realization of the proposed algorithm. Let parameters in (5) have the following form

For $0\le x<l-0$:

$l=1485m,\ c_1=331m/san,\ \rho_1=0.717kg/m^3,\ d_1=\sqrt{114^2-73^2}\cdot 10^{-3}m,\ \lambda_1=0.01;$

for $l+0<x\le 2l$:

$c_2=850m/san,\ \rho_2=700kg/m^3,\ d_2=0.073m,\ \lambda_2=0.23.$

$$A = \begin{bmatrix} 0 & -12252.2 & 0 & 0 \\ -4.05498 \cdot 10^{-6} & -2.9791131 & 0 & 0 \\ 0 & 1.162454 \cdot 10^{5} & 0 & -1.162454 \cdot 10^{5} \\ 2.818442 \cdot 10^{-6} & 0 & -2.818442 \cdot 10^{-6} & 1.795454 \cdot 10^{2} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 1.22522 \cdot 10^{4} \\ 4.054981 \cdot 10^{-6} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, V = \begin{bmatrix} 0 \\ 0 \\ 1.162454 \cdot 10^{7} \\ 2.8184422 \cdot 10^{-4} \end{bmatrix}.$$

After applying of the proposed algorithm we obtain that $\lambda_0 = 0.2373191$, $\lambda_1 = -15.19319$, here $\lambda_c = 0.2373198$ and $\tilde{\lambda}_c = 0.23$ ($\tilde{\lambda}_c$ the hydraulic resistance value from practice), Note that λ_c differs from $\tilde{\lambda}_c$ to the order10^{-3}, and it shows the efficiency of the proposed algorithm.

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REFERENCES

1. Aliev F. A., Hajiyeva N.S., Namazov A.A., Safarova N.A., Huseynova N.S. Asymptotical method to solution of identification problem for defining the parameters of discrete dynamical system in gas-lift process, Proceedings of the 6th international conference on control and optimization with industrial applications, Vol I, 6th International Conference on Control and Optimization with Industrial Applications (COIA), Baku, Azerbaijan, Jul 11-13, 2018, pp.55-57.
2. Aliev F.A., Aliev N.A., Safarova N.A., Tagiev R.M., Rajabov M.F. Sweep method for solving the Roesser type equation describing the motion in the pipeline, Applied Mathematics and Computation, V.295, 2017, pp.16-23.
3. Aliev F.A., Hajieva N.S., Namazov A.A., Safarova N.A.. Identification Problem for Determining the Parameters of a Discrete Dynamic System, International Applied Mechanics, V.55, N.1, 2019 , pp.110–116.
4. Aliev F.A., Hajiyeva N.S., Ismailov N.A., Mirsaabov S.M. Calculation algorithm defining the coefficient of hydraulic resistance on different areas of pump-compressor pipes in gas lift process by lines method, SOCAR Proceedings, N.4, 2019, pp.13-17.
5. Aliev F.A., Ilyasov M.Kh., Nuriev N.B. Problems of modeling and optimal stabilization of the gas lift process, International Applied Mechanics, V.46, N.6, 2010, pp.709-717.

6. Aliev F.A., Ismailov N.A., Haciyev H., Guliyev M.F. A method to determine the coefficient of hydraulic resistance in different areas of pump-compressor pipes, TWMS J. Pure Appl. Math., V.7, N.2, 2016, pp.211-217.
7. Aliev F.A., Ismailov N.A., Namazov A.A., Rajabov M.F. Algorithm for Calculating the Parameters of Formation of Gas-Liquid Mixture in the Shoe of Gas Lift Well, Appl. Comp. Math., V.15, N.3, 2016, pp.370-376.
8. Aliev F.A., Ismailov N.A., Namazov A.A., Rajabov M.F., Safarova N.A., Mamedova Y.V. Asymptotic Method for Solution of Identification Problem of the Nonlinear Dynamic Systems, Filomat, V.32, N.3, 2018, pp. 1025-1033.
9. Aliev F.A., Mutallimov M.M., Askerov I.M., Raguimov I.S. Asymptotic Method of Solution for a Problem of Construction of Optimal Gas-Lift Process Modes, Mathematical Problems in Engineering, V.2010, 2010, 10p.
10. Altshul A.D. Hydraulic Resistance, Moscow, Nedra, 1970.
11. Askerov I.M., Ismailov N.A. Asymptotic method for solution of the optimization problem with periodic boundary conditions and control in gaslift process, TWMS J. Pure Appl. Math., V.4, N.2, 2013, pp.237-241.
12. Benner Peter, Feng Lihong. Model order reduction for coupled problems (Survey), Appl. Comp. Math., V. 14, N. 1, 2015, pp.3-22.
13. Charniy I.A. Non-stationary motion of real fluid in pipes, Moscow , Nedra 1975.
14. El-Ganaini S.I.A., Mirzazadeh M., Biswas A. Solitons and Other Solutions to Long-Short Wave Resonance Equation, Appl. Comp. Math., V.14, N.3, 2015, pp.248-259.
15. Hajieva N.S., Namazov A.A., Askerov I.M., Maharramov I.A. Algorithm to solution of identification problem to determine the parameters of discrete dynamic system, Proceedings of IAM, V.5, N.2, 2016, pp.235-244.
16. Hajieva N.S., Safarova N.A., Ismailov N.A. Algorithm defining the hydraulic resistance coefficient by lines method in gas-lift process, Miskolc Mathematical Notes, V.18, N.2, 2017, pp.771-777.
17. Huseynova N.S., Orucova M.S., Safarova N.A., Hajiyeva N.S., Rustamova L.A. Numerical algorithms for solving the inverse problem. Proceedings of the 6th international conference on control and optimization with industrial applications, Vol I, 6th International Conference on Control and Optimization with Industrial Applications (COIA), Baku, Azerbaijan, Jul 11-13, 2018, pp.191-193.
18. Jbilou K. A Survey of Krylov-Based Methods for Model Reduction in Large-Scale MIMO Dynamical Systems, Appl. Comp.., V.15, N.2, 2016, pp.117-148.

19. Mirzadjanzadeh A.Kh., Akhmetov I.M., Khasaev A.M., Gusev V.I. Technology and technique of oil production, Moscow, Nedra,1986.
20. Mukhtarova N.S. Algorithm to solution the identification problem for finding the coefficient of hydraulic resistance in gas-lift process, Proceedings of IAM, V.4, N.2, 2015, pp.206-213.
21. Ryu Ernest K., Boyd Stephen. A Primer on Monotone Operator Methods (Survey), Appl. Comp. Math., V.15, N.1, 2016, pp.3-43.